

Random geometric phase sequence due to topological effects in our brane world from extra dimensions

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Abstract

Using Kaluza-Klein theory we discuss the quantum mechanics of a particle in the background of a domain wall (brane) embedded in extra dimensions. We show that the geometric phases associated with the particle depend on the topological features of those spacetimes. Using a cohomological modeling schema, we deduce a random phase sequence composed of the geometric phases accompanying the periodic evolution over the spacetimes. The random phase sequence is demonstrated some properties that could be experimental verification. We argue that it is related to the nonlocality of quantum entanglement.

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Quantum entanglement, one of the most fascinating and important features in quantum theory, is widely appreciated as an essential ingredient in quantum computations [1–4]. It demonstrates one of the most remarkable aspects of quantum theory is the incompatibility of quantum nonlocality with local-realistic theories [5]. Recently, an effective simulation of quantum entanglement using classical fields modulated with pseudorandom phase sequences is proposed [6]. Inspired by this, we argue that each quantum particle might be characterized by the wavefunction with a unique random phase sequence, namely the quantum particle might own some unknown intrinsic phase mechanism. The quantum scenario demonstrates quantum nonlocality i.e. violation of Bell’s inequality, free from “spooky action at a distance”.

As an alternative theory adopting gravity into quantum theory, superstring theory draws a lot of attention [7–10]. Different from other theories, superstring theory has perfect mathematical and physical structure, but hasn’t been experimentally validated directly for required unconceivable energy levels. Recently some researches have focused non-perturbative effects of extra dimensions, hoping it can be observed through the experiment. In Kaluza-Klein theory, the extra dimensions being rolled up to Planck length makes domain wall (brane) surrounded by particle string. This particle string winding movement around topological defects will bring non-perturbative geometric effects [11–14].

In quantum mechanics, the global phase of particle wavefunction has been ignored, due to the Born rule probability interpretation of a state vector at a single moment of time. The notion of a global geometric phase factor has been introduced in quantum mechanics by Berry [15, 16] and formulated in fiber bundle theoretic terms by Simon [17]. In 1959 Aharonov and Bohm showed that there exists the global geometric phase in the interference pattern of two coherent wavefunctions of a charged particle due to the existence of an electromagnetic field constrained within an infinitely long solenoid even the strength vanished [18]. Recently, Zafiris showed that a sheaf cohomological understanding of the origin of global geometric phase factors paves the way for understanding the global symmetry group of a quantum spectral beam [19]. They employ the local phase invariance of a quantum state vector is related to the observation of global gauge-invariant topological and geometric phase factors accompanying the periodic evolution over a space of control variables.

In this letter we will discuss the argument from two aspects: (1) what physical mechanisms to generate random phase sequences for quantum particles; (2) what verifiable theoretical

predictions deduced from the physical mechanisms. First we research geometric phases of particle string wound in the brane of extra dimensions based on the Kaluza-Klein theory. Then we research random phase sequences superposed of multiple geometric phases in the control variable space X of genus g in the framework of the cohomology theory proposed in Ref. [19]. Finally, we deduce three theoretical predictions that would be verified by future experiments.

In the conventional Kaluza-Klein approach the Universe has a topology $M_4 \otimes K$ where M_4 is our four dimensional Minkowski space and K is some compact manifold, with the volume typically set by a fundamental Planck length $l_{p_f} = 1/M_{P_f}$. Our world is a domain wall (brane) embedded in the extra-dimensional space with a rolled up dimension with size $R \sim l_{p_f}$. In this physical picture, the rolled extra dimension has one important difference from the particle theory version [7, 8]. A closed string can get wound several times around a rolled up the dimension. When a particle string does this, the particle string oscillations have a winding mode. The winding modes add a symmetry to the theory not present in particle physics.

Now we consider the geometric phase due to the winding modes [11–14]. The geometric phase is very similar to the Aharonov-Bohm phase of a charged particle traversing a loop including a magnetic flux [18], which is a nontrivial topological effect in a multiply connected space. According to Ref. [11], a toy model is to assume that $K = S_1$ with nontrivial closed curve homotopy class $H_1(S_1)$. This has the isometry group $G = U(1)$, broken by the brane down to identity. The brane position on S_1 is parameterized by one scalar modulus σ . This position can slowly vary with x_μ , the four-dimensional space-time point. Thus, the four-dimensional observer will perceive $\sigma(x_\mu)$ as a low-energy scalar field on M_4 . The particle string in the brane vibrates and periodically moves surrounding the extra dimensions. In the toy model, the target space of σ is obviously a circle S_1 , which the corresponding fundamental group $H_1(S_1) = \mathbb{Z}$. After a cyclic evolution of the environment parameters, the geometric phase can be expressed abstractly

$$\beta(C) = \oint_C \langle \phi | i \nabla \phi \rangle = i \int_0^{\sigma(\tau)=2\pi} \langle \phi(\sigma) | \frac{d}{d\sigma} | \phi(\sigma) \rangle d\sigma \quad (1)$$

where σ and τ are coordinates on the brane representing space and time along the string. The non-integrable geometric phase $\beta(C)$ tells us something about the geometry of the circuit and about regions of the brane characterizing (for example, enclosed by) the circuit.

Whereas $\beta(C)$ is the holonomy due to parallel transport around a circuit, in analogy to the electromagnetic Aharonov-Bohm effect [12]. The geometry of the brane is important in Berry's formulation because geometric and topological features can present obstructions to a global definition of the string phases [20]. This is why the geometric phase is independent on the size of the extra-dimension even rolled up in a circle of radius $R \sim l_{pf}$. We note again that the time dependence of geometric phase $\beta(C)$ is implicitly introduced via time-parameterized paths on K , and $\beta(C) = \beta(T)$ during the string vibrating within a finite temporal period T . We obtain the geometric phase $\beta(\tau + nT) = n\beta(T) \delta(\tau + nT)$, where n is an integer, the winding number. In conclusion, the geometric phase must be periodically appeared if the string vibrating without any disturbance.

Superstring theory dictates that the universe could be a topology $M_4 \otimes K$ with extra space K with multiple dimensions, which could be a Calabi-Yau manifold. In Ref. [19], a sheaf-cohomological concept is introduced to research the relation to the observation of global gauge-invariant topological and geometric phase factors accompanying the periodic evolution over a space X of control variables. We consider the parametric dependence of the particle string vibrating in the brane is analogy to the control variable space X and the notion of a vector sheaf generalizes as Ref. [19]. Considering the homology group $H_1(X)$ is composed of 1-dimensional homology class in X space, the winding path σ of the string in the brane is isomorphism of homology cycle $\gamma \in H_1(X)$. According Ref. [19], a cohomology class in the cohomology group $H^1(X, U(1))$ can be evaluated at the homology cycle γ by means of the pairing:

$$H_1(X) \times H^1(X, U(1)) \rightarrow U(1) \quad (2)$$

to obtain a global observable gauge-invariant geometric phase factor $\beta \in U(1)$.

Further, we consider X space is a compact Riemann surface of genus g . Using simplicial complex analysis, we obtain a typical base $[\gamma_1], [\gamma_2], \dots, [\gamma_{2g}]$ of $H_1(X)$ composed of all homology classes and the dimension $2g$ of cohomology group $H^1(X, U(1))$ [21]. We assume the winding path σ of the particle string is a simplicial 1-chain is a formal sum of 1-cycles:

$$\sigma = \sum_{i=1}^{2g} m_i \gamma_i \quad (3)$$

where $m_i \in \mathbb{Z}$. Based on the previous analysis, we assume that the string obtain geometric

phases β_i in each homology cycle $\gamma_i \in H_1(X)$. The total geometric phase can be express:

$$\beta(\tau) = \sum_{i=1}^{2g} \beta_i(\tau + m_i T_i) = \sum_{i=1}^{2g} m_i \beta_i(T_i) \delta(\tau + m_i T_i) \quad (4)$$

where m_i is the winding number. The geometric phases β_i and periods T_i of the particle string sweeping each homology cycle γ_i should be different. According to the almost-periodic theory, the total geometric phase is almost-periodic if any two periods of geometric phases are incommensurable [22, 23]. As a result, the total geometric phase becomes an almost-periodic random sequence [24–26].

According the qualitative analysis of the superposed geometric phase, we propose three theoretical predictions:

Superposition: The geometric phase of quantum particle due to topological defects of extra-dimensions is a phase sequence superposed of multiple periodic functions, and the maximum count of the periodic functions should be twice of genus g of X space.

Almost-periodic: The geometric phase sequence is almost-periodic.

Random: The geometric phase sequence is an almost-periodic random sequence.

In Ref. [6], an effective simulation of quantum entanglement using classical fields modulated with pseudorandom phase sequences is proposed. Consider the simplest case that the mode $|1\rangle$ of the classical fields $|\psi_a\rangle$ and $|\psi_b\rangle$ are exchanged by a mode exchanger constituted by mode splitters and combiners as shown in Ref. [6]. When the modes are exchanged, the phase sequences modulated on classical fields also are exchanged. The classical fields efficiently simulate two entangled particles. However, it will be rather complicated for quantum particles, which involves particle string interaction in the framework of superstring theory [7, 8]. Consider the simple case, the base of homology group $H_1(X)$ and the winding modes of two particle strings are exchanged through some string interactions. As a result, the random geometric phase functions (sequences) $\beta^{(a)}(\tau)$ and $\beta^{(b)}(\tau)$ of the superposition states are also exchanged similar to the classical simulation:

$$\begin{aligned} |\psi'_a\rangle &= \frac{e^{i\beta^{(a)}(\tau)}}{\sqrt{2}} \left(|0\rangle_a + e^{i\gamma^{(a)}(\tau)} |1\rangle_b \right) \\ |\psi'_b\rangle &= \frac{e^{i\beta^{(b)}(\tau)}}{\sqrt{2}} \left(|0\rangle_b + e^{i\gamma^{(b)}(\tau)} |1\rangle_a \right) \end{aligned} \quad (5)$$

where $\gamma^{(a)}(\tau) = -\gamma^{(b)}(\tau) = \beta^{(b)}(\tau) - \beta^{(a)}(\tau)$ is also an almost-periodic random function. We obtain the results of the particles in the measurement $P(\theta_a, \tau) = \cos(\theta_a, \gamma^{(a)}(\tau))$ and

$P(\theta_b, \tau) = \cos(\theta_b, \gamma^{(b)}(\tau))$. Then we define the temporal correlation function:

$$E(\theta_a, \theta_b; t) = \frac{1}{C} \int_0^t P(\theta_a, \tau) P(\theta_b, \tau) d\tau = \cos(\theta_a + \theta_b) + \frac{1}{C} \int_0^t \cos(\theta_a - \theta_b + 2\gamma^{(a)}(\tau)) d\tau \quad (6)$$

where C is the normalized coefficient. According to Riesz-Fischer theorem, the correlation function $E(\theta_a, \theta_b; t)$ can be related with the trigonometric polynomials and Fourier analysis [22]. Due to the almost-periodic random of $\gamma^{(a)}(\tau)$, we obtain

$$\lim_{t \rightarrow \infty} \frac{1}{C} \int_0^t \cos(\theta_a - \theta_b + 2\gamma^{(a)}(\tau)) d\tau \rightarrow 0 \quad (7)$$

and $E(\theta_a, \theta_b) = \cos(\theta_a + \theta_b)$ demonstrates violation of Bell's inequality for two quantum particles. The correlation function $E(\theta_a, \theta_b; t)$ should be a useful tool to analysis the motion of particle strings in the brane.

Long term since, quantum mechanics suffers from the complete and nonlocality doubts. There might be two long-standing beliefs for this: (1) the randomness of quantum only caused by the measurement process; (2) the neglect of the overall phase of wavefunction due to no contribution to the probability distribution. Based on the previous analysis, we propose a new phase mechanism to explain the nonlocality of quantum mechanics. The randomness of quantum might be originated from the random geometric phase sequence of particle string wound in the brane of extra dimensions. The random geometric phase sequence due to the topological features of X space might be not only confined between different particles also as a relative phase sequence between different eigenstates of the measurement operators, which also results in the randomness in quantum measurement of a single particle. It is very important how to verify these predictions in the experiment. We believe Bose-Einstein condensation [27] might be a good experiment demonstration because the random geometric phase sequence should be amplified to macroscopic quantum phenomena.

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